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Aggressive Longitudinal Aircraft Trajectory Tracking Using Nonlinear Control

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Flight-control system designs are complicated if the aircraft dynamics are nonlinear and nonminimum phase. The nonminimum phase property can result from the choice of output vector and coupling between the moment generating actuators and the aerodynamic forces on the aircraft. In this paper we study a flight-control problem for a conventional aircraft longitudinal dynamic model that explicitly includes the coupling between the moment generating actuators and the aerodynamic forces. In particular, we study the execution of a maneuver for which the aircraft is intended to track a given motion in a vertical plane. We formulate the problem as a nonlinear tracking control problem. Controllers are developed for an aggressive maneuver that requires the use of a two-degrees-of-freedom controller design. We demonstrate the value of this control architecture in order to achieve aggressive maneuvering with good tracking performance. Our approach throughout is to make use of nonlinear control theory. Our analysis is complicated by the nonminimum phase characteristics of the flight model.

Introduction

HIGHLY maneuverable aircraft provide examples of nonlinear nonminimum phase dynamic systems. The nonminimum phase property is a result of aerodynamic body forces that are directly produced by aerodynamic control surfaces and of the choice of tracking outputs.

In recent years nonlinear decoupling theory and dynamic inversion approaches[1,2] have been applied to design flight-control systems. However, it has been shown that straightforward application of inversion approaches to nonlinear nonminimum phase flight-control models can result in a system with a linear input/output response but unstable zero dynamics.[3–5] In-flight[6–8] controls are designed using nonlinear inversion, but no attempt is made to investigate the stability of the resulting zero dynamics.

Progress has been made in[3] and[5] in which a weakly nonminimum phase system is approximated by a minimum phase one. Recent progress in output tracking of nonlinear systems has been achieved based on development of an output regulation theory.[9] Output regulation ensures internal stability with asymptotic output tracking for a class of nonlinear systems but requires solving a set of partial differential equations, and it is limited to reference trajectories generated by an exosystem.

Unlike previous work on flight-control problems where the main emphasis was on stabilization or tracking constant commands, this paper addresses position tracking for flight vehicle models that can be characterized as multi-input, multi-output, nonlinear, nonminimum phase systems. The objective is to design a tracking control law such that outputs of the closed-loop system satisfy certain maneuver performance objectives with internal stability.

In this paper we develop a nonlinear tracking controller for a conventional fixed-wing aircraft. The flight model provides a challenging example for nonlinear flight-control studies. The model captures the essential features of the longitudinal dynamics of a conventional fixed-wing aircraft.

Our approach is based on a system decomposition, after suitable state and control coordinate transformations, into two parts: a linear input-output subsystem with trivial internal dynamics (the minimum phase part) and an input-output subsystem with unstable internal dynamics (the nonminimum phase part). A requirement for this decomposition to be effective is that the nonminimum phase part is stabilizable in the first approximation. The tracking problem, in error coordinates, can be treated as a stabilization problem of a nominal time-invariant system perturbed by terms that depend on the tracking commands and their derivatives. These perturbation terms appear only in the internal dynamics of the nonminimum phase part. A feedback control law, which solves the original tracking problem, is then designed in two parts. First, a fast stabilizing control law is designed based on dynamic inversion for the minimum phase part. Second, a robust locally stabilizing linear feedback control law is designed for the nonminimum phase part. The robust controller for the nonminimum phase part provides a tradeoff between the class of commands that can be tracked and the achievable performance of the closed loop.

For linear nonminimum phase systems with feedback control only, there exist fundamental limitations in the achievable performance[10,11] of the closed-loop system. Thus, for nonlinear nonminimum phase systems we expect that feedback control can only track trimmed and near trimmed commands (nonaggressive tracking). To solve the aggressive trajectory tracking problem, we modify the control design for the nonminimum phase part by including a feedforward control part as well as a feedback control part. The feedforward control is computed using an iterative numerical procedure called noncausal stable inversion.[12] The use of feedforward control is essential to achieve good tracking performance when dealing with nonminimum phase systems.

There is a large literature that illustrates the application of various nonlinear control approaches to various flight control problems. There are fewer papers that treat output tracking for flight control problems, see, for example, Ref. 13. One widely studied problem is the planar vertical takeoff and landing problem, see, for example, Refs. 4, 14, and 15. The present paper represents a nontrivial extension of our paper[16] treating the approximated longitudinal aircraft model introduced by Tomlin et al. in Ref. 17. The present paper also introduces a new nonlinear control design approach that, we believe, leads to improved closed-loop properties, especially in performing aggressive flight maneuvers.

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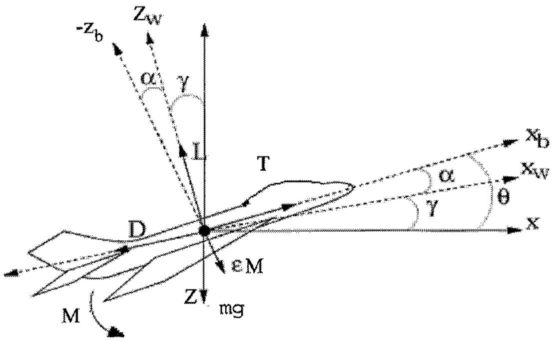

Fig. 1 Longitudinal aircraft model in flight.

Equations of Motion of the Flight Vehicle

We develop a nonlinear model, which describes the longitudinal dynamics of an aircraft in forward flight. The longitudinal aircraft model provides a challenging example for nonlinear flight-control studies. The aircraft model includes aerodynamic forces as well as coupling between the aerodynamic pitch moment and the aerodynamic translational forces. Figure 1 shows a prototype longitudinal aircraft in flight. The aircraft state is the position X, Z of the aircraft center of mass, the pitch angle θ of the aircraft, and the corresponding velocities $\dot{X}$, $\dot{Z}$, $\dot{\theta}$. The control inputs T and M are, respectively, the thrust along the aircraft body fixed x axis and the pitching moment about the aircraft center of mass.

Deflecting an elevator upward produces a small negative lift force which generates a positive pitching moment about the center of mass of the aircraft. The presence of this parasitic aerodynamic force makes the longitudinal aircraft model nonminimum phase. In this case nonlinear control design method such as dynamic inversion is not directly applicable to this flight-control problem.[18] In this section we provide detailed expressions for the effects of the aerodynamic pitch moment on the aerodynamic translational forces. We express the aerodynamic lift and drag in terms of the aerodynamic pitch moment.

The full longitudinal equations of motion of an aircraft can be written as

$$m\ddot{X} = -D\cos\gamma - L\sin\gamma + T\cos\theta \tag{1}$$

$$-m\ddot{Z} = D\sin\gamma - L\cos\gamma - T\sin\theta + mg \tag{2}$$

$$I_y\ddot{\theta} = M \tag{3}$$

The aerodynamic lift force L and the aerodynamic drag force D, which are functions of the aerodynamic pitching moment M,[19] are given by

$$L = Q_s(\tilde{C}_{l0} + \tilde{C}_{l\alpha}\alpha) - mgK_1\cos\gamma - K_1T\sin\alpha - K_2V\dot{\theta} + \epsilon_0 M \tag{4}$$

$$D = Q_s\left[C_{d0} + \left(\mathcal{K}\big/Q_s^2\right)L^2\right] \tag{5}$$

where

$$\epsilon_0 = C_{l\delta_e}\big/\bar{c}\left(C_{m\delta_e} - \bar{c}C_{l\delta_e}C_{m\dot{\alpha}}\rho S\big/4m\right), \qquad K_1 = \epsilon_0\bar{c}^2C_{m\dot{\alpha}}\rho S\big/4m$$

$$K_2 = \epsilon_0\bar{c}^2\rho S(C_{mq} + C_{m\dot{\alpha}})\big/4, \qquad \tilde{C}_{l0} = (1 + K_1)C_{l0} + \epsilon_0\bar{c}C_{m0}$$

$$\tilde{C}_{l\alpha} = (1 + K_1)C_{l\alpha} + \epsilon_0\bar{c}C_{m\alpha}$$

Equations (4) and (5) show the relation between the aerodynamic lift L and drag D and the aerodynamic pitching moment M. The parameter ϵ_0 in Eq. (4) gives the explicit coupling between the aerodynamic forces and the aerodynamic control moment. This parameter represents approximately the ratio between the aerodynamic lift force and the aerodynamic moment generated by the elevator. We scale the longitudinal aircraft model by dividing Eqs. (1) and (2) by mg and dividing Eq. (3) by I_y. Define $x = X/g$, $z = -Z/g$, $u_x = T/(mg)$, $u_m = M/I_y$, and $\epsilon = \epsilon_0 I_y/(mg)$. Then the rescaled dynamics become

$$\ddot{x} = -D'\cos\gamma - L'\sin\gamma + u_x\cos\theta \tag{6}$$

$$\ddot{z} = -D'\sin\gamma + L'\cos\gamma + u_x\sin\theta - 1 \tag{7}$$

$$\ddot{\theta} = u_m \tag{8}$$

where L' and D' are dimensionless lift and drag forces given by

$$L' = a_l v^2(1 + c\alpha) - K_1\cos\gamma - (K_2/m)v\dot{\theta} + \epsilon u_m - K_1u_x\sin\alpha \tag{9}$$

$$\begin{aligned} D' = {} & a_d v^2[1 + b(1 + c\alpha)^2] + \left(2K_3\big/v^2\right)\left[a_l v^2(1 + c\alpha)\right. \\ & \left. - K_1\cos\gamma - (K_2/m)v\dot{\theta}\right](\epsilon u_m - K_1u_x\sin\alpha) \\ & + \left(K_3\big/v^2\right)\left\{(\epsilon u_m - K_1u_x\sin\alpha)^2 + [K_1\cos\gamma + (K_2/m)v\dot{\theta}]^2\right. \\ & \left. - 2a_l v^2(1 + c\alpha)[K_1\cos\gamma + (K_2/m)v\dot{\theta}]\right\} \end{aligned} \tag{10}$$

where

$$v = \frac{V}{g}, \qquad a_l = \frac{\rho g S\tilde{C}_{l0}}{2m}, \qquad a_d = \frac{\rho g S C_{d0}}{2m}$$

$$c = \frac{\tilde{C}_{l\alpha}}{\tilde{C}_{l0}}, \qquad b = \frac{\mathcal{K}\tilde{C}_{l0}{}^2}{C_{d0}}, \qquad K_3 = \frac{2m\mathcal{K}}{\rho g S}$$

The longitudinal aircraft model is not linear affine in the control, and this is obvious from the expressions for the aerodynamic lift and drag forces in Eqs. (9) and (10). Define new control variables (v_x, v_z) in terms of (u_x, u_m) by the following invertible transformation:

$$v_x = -D'\cos\gamma - L'\sin\gamma + u_x\cos\theta \tag{11}$$

$$v_z = -D'\sin\gamma + L'\cos\gamma + u_x\sin\theta - 1 \tag{12}$$

Using Eqs. (9), (11), and (12), we obtain

$$\begin{aligned} \epsilon u_m = {} & (1 + K_1)u_x\sin\alpha + (v_z + K_1 + 1)\cos\gamma \\ & - v_x\sin\gamma - a_l v^2(1 + c\alpha) \end{aligned} \tag{13}$$

By using the relation $D' = a_d v^2 + (K_3/v^2)L'^2$ with Eqs. (11–13), we obtain after some algebraic manipulations

$$\delta(\bar{x})u_x^2 + b(\bar{x}, \bar{v})u_x + c(\bar{x}, \bar{v}) = 0 \tag{14}$$

where $\bar{x} = (x, \dot{x}, z, \dot{z}, \theta, \dot{\theta})$, $\bar{v} = (v_x, v_z)$, and

$$\delta(\bar{x}) = K_3(\sin\alpha/v)^2\cos\alpha$$

$$b(\bar{x}, \bar{v}) = \left(K_3\big/v^2\right)\sin 2\alpha[(v_z + 1)\cos\gamma - v_x\sin\gamma] - \cos^2\alpha$$

$$\begin{aligned} c(\bar{x}, \bar{v}) = {} & \left(K_3\big/v^2\right)[(v_z + 1)\cos\gamma - v_x\sin\gamma]^2\cos\alpha \\ & + [(v_z + 1)\cos\gamma - v_x\sin\gamma]\sin\alpha + (v_z + 1)\sin\theta \\ & + v_x\cos\theta + a_d v^2\cos\alpha \end{aligned}$$

The output tracking control problem is defined in terms of specified command functions

$$x_c = X_c/g, \qquad z_c = Z_c/g$$

which are assumed to be twice differentiable.

Using the preceding control transformation, Eqs. (6–8) can be written in the error dynamics normal form as

$$\ddot{e}_x = \tilde{v}_x \tag{15}$$

$$\ddot{e}_z = \tilde{v}_z \tag{16}$$

$$\begin{aligned} \epsilon\ddot{\theta} = {} & (1 + K_1)u_x\sin\alpha - a_l\left[(\dot{e}_x + \dot{x}_c)^2 + (\dot{e}_z + \dot{z}_c)^2\right](1 + c\alpha) \\ & + (\tilde{v}_z + \ddot{z}_c + K_1 + 1)\cos\gamma - (\tilde{v}_x + \ddot{x}_c)\sin\gamma \end{aligned} \tag{17}$$

where $e_x = x - x_c$, $e_z = z - z_c$, $\tilde{v}_x = v_x - \ddot{x}_c$, and $\tilde{v}_z = v_z - \ddot{z}_c$.

The zero dynamics of the longitudinal aircraft system can be obtained by assuming that the aircraft is flying with a constant commanded horizontal speed $\dot{x}_c^*$ at a constant altitude so that $\dot{z}_c = 0$. In this case $\ddot{x} = \ddot{z} = \dot{z} = \gamma = 0$ and $\alpha = \theta$. Without lost of generality, we linearize Eq. (17) around $\theta = \dot{\theta} = 0$ to obtain the zero dynamics equation:

$$\ddot{\theta} = (1/\epsilon)\left[(1 + K_1)\left(K_3/\dot{x}_c^{*2}\right) + (a_d - a_l c)\dot{x}_c^{*2}\right]\theta \tag{18}$$

For conventional aircraft $[(1 + K_1)(K_3/\dot{x}_c^{*2}) + (a_d - a_l c)\dot{x}_c^{*2}]$ and ϵ are negative constants. Hence the origin of Eq. (18) is an unstable saddle equilibrium, which indicates that the longitudinal aircraft system is nonminimum phase.

Control System Design

We consider the longitudinal aircraft model given by Eqs. (15–17). The driven dynamics (17) is time invariant when $\ddot{x}_c = \ddot{z}_c = 0$. All commands for which the driven dynamics is time invariant are trimmed commands, and accordingly we define all trajectories that are generated by trimmed commands as trimmed trajectories. On the other hand, all trajectories that are generated by nontrimmed commands are called nontrimmed trajectories. A nontrimmed trajectory can be either aggressive or nonaggressive. Nonaggressive trajectories are those generated by small $\ddot{x}_c$ and $\ddot{z}_c$ commands. Aggressive trajectories are those trajectories that correspond to either large acceleration commands or small acceleration commands but with high tracking accuracy requirement.

Tracking Control System Design Using Feedback

Minimum Phase/Nonminimum Phase Decomposition

We treat the longitudinal aircraft system (15–17), as an interconnection of two subsystems; a minimum phase part defined by the horizontal flight dynamics

$$\ddot{e}_x = \tilde{v}_x \tag{19}$$

and a nonminimum phase part defined by the vertical and pitching flight dynamics

$$\ddot{e}_z = \tilde{v}_z \tag{20}$$

$$\epsilon\ddot{\theta} = (1 + K_1)u_x \sin\alpha - a_l\left[(\dot{e}_x + \dot{x}_c)^2 + (\dot{e}_z + \dot{z}_c)^2\right](1 + c\alpha) + (\tilde{v}_z + \ddot{z}_c + K_1 + 1)\cos\gamma - (\tilde{v}_x + \ddot{x}_c)\sin\gamma \tag{21}$$

It is clear that there is one-way coupling between the two subsystems, namely, from the minimum phase part to the nonminimum phase part. The driven dynamics of Eq. (21) can be written as

$$\dot{\eta} = f(\eta, \dot{e}_x, \dot{e}_z, \tilde{v}_x, \tilde{v}_z, Y_c) \tag{22}$$

where $\eta = (\theta, \dot{\theta})$ and $Y_c = (\dot{x}_c, \dot{z}_c, \ddot{x}_c, \ddot{z}_c)$. Suppose we write Eq. (22) as

$$\dot{\eta} = A_{11}\bar{\eta} + A_{12}\bar{e}_z + B_{11}\tilde{v}_z + g(\eta, \dot{e}_x, \dot{e}_z, \tilde{v}_x, \tilde{v}_z, Y_c) \tag{23}$$

where

$$\bar{e}_z = (e_z, \dot{e}_z), \qquad A_{11} = \frac{\partial f}{\partial \eta}\left(\eta^*, 0, 0, 0, 0, Y_c^*\right)$$

$$A_{12} = \frac{\partial f}{\partial \bar{e}_z}\left(\eta^*, 0, 0, 0, 0, Y_c^*\right), \qquad B_{11} = \frac{\partial f}{\partial v_z}\left(\eta^*, 0, 0, 0, 0, Y_c^*\right)$$

$$g(\eta, \dot{e}_x, \dot{e}_z, \tilde{v}_x, \tilde{v}_z, Y_c) = f(\eta, \dot{e}_x, \dot{e}_z, \tilde{v}_x, \tilde{v}_z, Y_c) - A_{11}\bar{\eta} - A_{12}\bar{e}_z - B_{11}\tilde{v}_z$$

Here $Y_c^* = (\dot{x}_c^*, 0, 0, 0)$, $\bar{\eta} = \eta - \eta^*$, and $\eta^* = (\theta^*, 0)$, where θ^* is an equilibrium pitch angle satisfying

$$f\left(\theta, 0, 0, 0, 0, Y_c^*\right) = 0$$

Therefore, the nonminimum phase dynamics described by Eqs. (20) and (21) can be viewed as a perturbation of the nominal system

$$\ddot{e}_z = \tilde{v}_z \tag{24a}$$

$$\dot{\eta} = A_{11}\bar{\eta} + A_{12}\bar{e}_z + B_{11}\tilde{v}_z \tag{24b}$$

with perturbation $g(\eta, \dot{e}_x, \dot{e}_z, \tilde{v}_x, \tilde{v}_z, Y_c)$.

Control of the Minimum Phase Dynamics

Based on the preceding decomposition, feedback inversion can be used to solve the output tracking problem for the minimum phase dynamics (19). We choose a high gain controller

$$\tilde{v}_x = -(\beta_1/\epsilon_1)\dot{e}_x - \left(\beta_2/\epsilon_1^2\right)e_x \tag{25}$$

where $\epsilon_1 > 0$ is a timescale parameter and $\beta_i, i = 1, 2$ are constants to be chosen such that the closed-loop system is exponentially stable. If $\beta_i > 0$, $i = 1, 2$, the closed-loop minimum phase part is exponentially stable for any $\epsilon_1 > 0$, and hence for any differentiable output command x_c, $x - x_c \to 0$ as $t \to \infty$.

Control of the Nonminimum Phase Dynamics

Following the development in Refs. 19–21 we use an linear quadratic regulator (LQR) approach to design a stabilizing feedback law for the nominal system (24):

$$\tilde{v}_z = -k_1 e_z - k_2\dot{e}_z - k_3(\theta - \theta^*) - k_4\dot{\theta} \tag{26}$$

where $[k_1, k_2, k_3, k_4] = R^{-1}B^TP$ and P is the solution of the algebraic Riccati equation

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{27}$$

where Q is positive definite matrix, $R > 0$, $B = (0, 1, 0, 1)$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{\partial f}{\epsilon\partial \dot{z}}\left(\eta^*, 0, 0, 0, 0, Y_c^*\right) & \dfrac{\partial f}{\epsilon\partial \theta}\left(\eta^*, 0, 0, 0, 0, Y_c^*\right) & 0 \end{bmatrix}$$

Tracking Control Result

We now substitute the controllers given by Eqs. (25) and (26) into Eqs. (15), (16), and (23) to obtain the exact closed loop:

$$\ddot{e}_x = -(\beta_1/\epsilon_1)\dot{e}_x - \left(\beta_2/\epsilon_1^2\right)e_x \tag{28}$$

$$\dot{\hat{z}} = A_c\hat{z} + \hat{g}(e_x, \dot{e}_x, \hat{z}, Y_c) \tag{29}$$

Here $\hat{z} = (e_z, \dot{e}_z, \eta)$ and $A_c = A - BK$, where $K = [k_1, k_2, k_3, k_4]$.

Assumption 1: The matrix A_c is Hurwitz.

Assumption 2: The perturbation term $\hat{g}(e_x, \dot{e}_x, \hat{z}, Y_c)$ satisfies the following inequality on $\|\hat{z}\|_2 < r$:

$$\|\hat{g}(0, 0, \hat{z}, Y_c)\| \le \gamma(Y_c)\|\hat{z}\| + \delta(Y_c) \tag{30}$$

where $\gamma[Y_c(t)]: R \to R$ and $\delta[Y_c(t)]: R \to R$ are nonnegative, continuous functions that satisfy the following inequalities:

$$\int_0^t \gamma[Y_c(\tau))\,\mathrm{d}\tau \le kt + c, \qquad t \ge 0$$

$$\sup_{t\ge 0}\delta[Y_c(t)] < \frac{\lambda_{\min}(P)\alpha r}{\lambda_{\max}(P)\rho}$$

for some nonnegative constants k, c, α, and ρ, where

$$k < \frac{\lambda_{\min}(P)\lambda_{\min}(\hat{Q})}{2\lambda_{\max}^2(P)}, \qquad \alpha = \frac{1}{2}\left[\frac{\lambda_{\min}(\hat{Q})}{\lambda_{\max}(P)} - k\frac{2\lambda_{\max}(P)}{\lambda_{\min}(P)}\right] > 0$$

$$\rho = \exp\left[\frac{\lambda_{\max}(P)c}{\lambda_{\min}(P)}\right] \ge 1$$

where $\hat{Q} = Q + PBR^{-1}B^TP$.

We use Assumptions 1 and 2 for the remainder of this paper. We consider two cases for which conclusions can be made about the closed-loop system defined by Eqs. (28) and (29):

1) Assume the horizontal acceleration commanded $\ddot{x}_c = 0$ and the vertical acceleration command $\ddot{z}_c = 0$. There exists ϵ_1^* such that if $0 < \epsilon_1 < \epsilon_1^*$ then the origin is an asymptotically stable equilibrium of the closed-loop defined by Eqs. (28) and (29). The origin corresponds to exact output tracking; and for any initial state sufficiently close to the origin, $x - x_c \to 0$, and $z - z_c \to 0$ as $t \to \infty$.

2) Assume the commanded vertical position z_c and the commanded horizontal position x_c are such that the following robust stability inequalities are satisfied on $\|\hat{z}_1 - \hat{z}_2\|_2 < r$:

$$\|\hat{g}(0, 0, \hat{z}_1, Y_c) - \hat{g}(0, 0, \hat{z}_2, Y_c)\| \leq \gamma(Y_c)\|\hat{z}_1 - \hat{z}_2\|_2 \quad (31)$$

$$\|\hat{g}(e_x, \dot{e}_x, \hat{z}, Y_c) - \hat{g}(0, 0, \hat{z}, Y_c)\| < (c_2/c_1)\|(e_x, \dot{e}_x)\|_2 \quad (32)$$

Then $x - x_c \to 0$ as $t \to \infty$, and the tracking error $z - z_c$ is uniformly ultimately bounded with ultimate bound

$$b > \frac{\lambda_{\max}(P)\rho\beta}{2\lambda_{\min}(P)}$$

Proofs of these results follow from Refs. 19–21. The preceding bounds on the vertical and horizontal acceleration commands are conservative as a consequence of a worst-case analysis.

Tracking Simulations Using Feedback

We consider a DC8 aircraft to illustrate the preceding control design for the longitudinal aircraft model. The controller is designed for a nominal value of $\epsilon = 0.23$, which corresponds to realistic aerodynamic data.[22] The controller parameters $R = 0.1$, $Q = \mathrm{diag}(0.5, 5, 1, 1)$, $\beta_1 = 0.01$, $\beta_2 = 0.2$, and $\epsilon_1 = 0.1$.

Figure 2 shows the total error in the aircraft response to a step command from 100 to 130 m/s for the horizontal velocity and to a vertical position command corresponding to a nap of the Earth (NOE) maneuver given by $z_c = (\frac{250}{2}) * [1 - \cos(\pi * t/60)](m)$; Figure 2 shows the total error over the maneuver period of 120 s. It can be seen that the total error is not zero. This is expected because we have shown that exact tracking is not possible if the vertical or horizontal acceleration is not zero. Figure 3 shows the control signals required to execute the NOE maneuver only over the initial transient period of 2 s. In the following sections we consider tracking the same NOE maneuver, but the objective this time is to achieve exact tracking or zero error tracking with internal stability. Thus we will consider an aggressive trajectory control design approach to achieve such objectives.

Tracking Control System Design Using Feedback and Feedforward

An aggressive flight maneuver occurs when the maneuver corresponds to large vehicle acceleration commands or when exact tracking is required. In this case the near-trimmed flight controllers that are based on static state feedback can be expected to fail or to lead to poor tracking performance. To solve the aggressive trajectory tracking problem, we must modify the nonminimum phase control design by including feedforward control as well as a feedback control. We show that this controller leads to asymptotic tracking of aggressive commands with closed-loop stability.

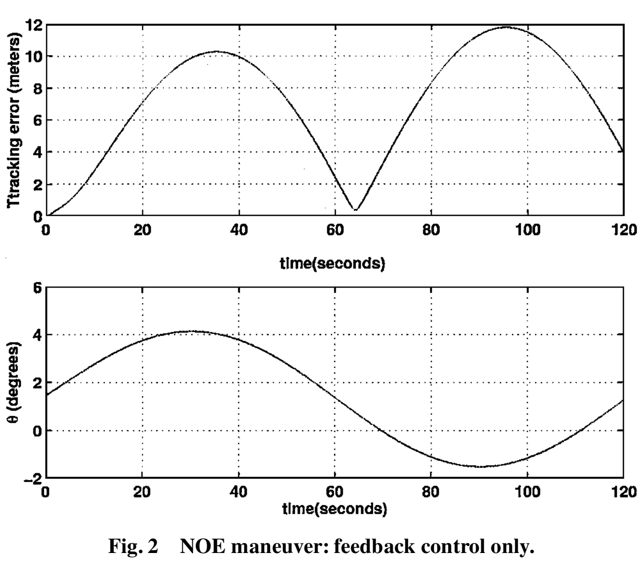

Fig. 2 NOE maneuver: feedback control only.

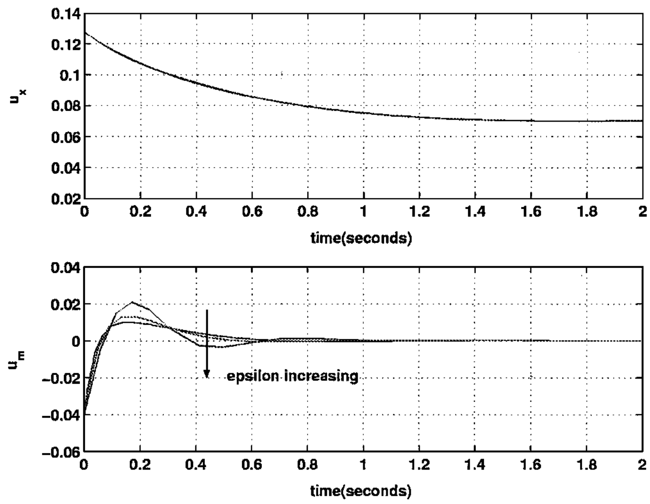

Fig. 3 Control responses: feedback control only.

Our Conceptual Approach to Tracking Aggressive Maneuvers

In the case of nonaggressive trajectory tracking design, the nonminimum phase controller uses feedback of the states of the internal dynamics in order to stabilize the zero dynamics. For nonaggressive commands the inputs (output commands and derivatives) to the driven dynamics are small, and hence the states of the driven dynamics remain bounded.

For aggressive maneuvers the inputs to the driven dynamics are large, and in this case the feedback controller is often unable to maintain bounded driven dynamics. This in turn can lead to instability. To overcome this problem, the feedforward control counterbalances the large perturbations induced by the aggressive commands. The overall control design approach for aggressive tracking has the merit that it results in a tracking controller with feedforward terms and constant gain feedback terms.

Our approach in solving the tracking problem is to compute first a bounded feedforward control using the noncausal stable inversion approach.[12] The feedback controller is then designed using a modified LQR approach with singular perturbation approach that allows design of a high gain controller for the minimum phase part and an LQR controller for the nominal nonminimum phase part (Fig. 4). The LQR/singular perturbation method allows us to study the stability of the nonminimum phase part independently of the minimum phase dynamics.

It is important to emphasize that in our approach the noncausal stable inversion method is used after the original flight equations are transformed to normal form, expressed in terms of error coordinates and decomposed into a minimum phase part and a nonminimum phase part. In this format the noncausal inversion method is only applied to the nonminimum phase part, which is of lower dimension than the original nonminimum phase problem. Also our approach avoids the complexity of using time-varying feedback gains.

We consider the flight control problem where the aircraft performs a NOE maneuver in a vertical plane. Previously we considered the same maneuver, but we used a trimmed flight controller to track a nontrimmed trajectory. The result of the trimmed tracking controller as seen in Fig. 2 is not satisfactory because the tracking errors are relatively large.

In this section we use the noncausal stable inversion approach to compute a feedforward control term that enhances the performance of the closed-loop system. It is important to mention that the aircraft flight model is not linear affine in the control. This makes the computation of the feedforward control numerically more challenging.

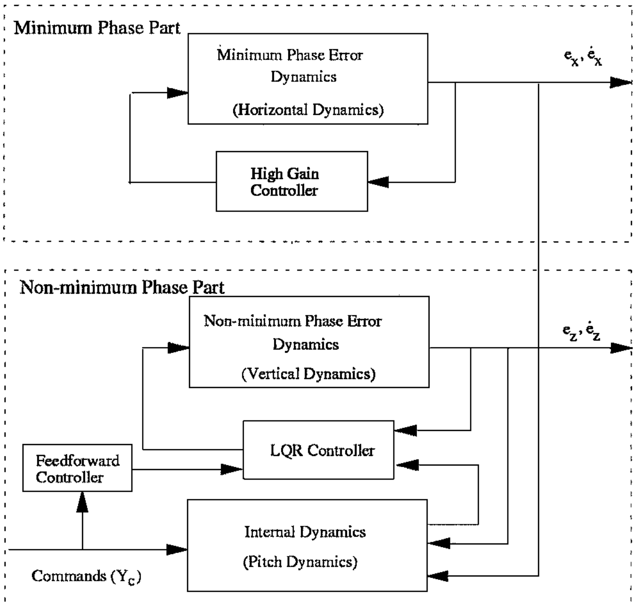

Fig. 4 Block diagram of the closed loop.

The closed-loop system defined by Eqs. (28) and (29), in terms of the original states, is given by

$$\ddot{x} = v_x \tag{33}$$

$$\ddot{z} = v_z \tag{34}$$

$$\epsilon\ddot{\theta} = (1 + K_1)u_x \sin\alpha + (v_z + K_1 + 1)\cos\gamma - v_x \sin\gamma - a_l v^2(1 + c\alpha) \tag{35}$$

where

$$v_x = \ddot{x}_c - (\beta_1/\epsilon_1)(\dot{x} - \dot{x}_c) - \left(\beta_2/\epsilon_1^2\right)(x - x_c) \tag{36}$$

$$v_z = \ddot{z}_c - k_1(z - z_c) - k_2(\dot{z} - \dot{z}_c) - k_3(\theta - \theta^*) - k_4\dot{\theta} \tag{37}$$

Here $\beta_1 > 0$, $\beta_2 > 0$, and $[k_1, k_2, k_3, k_4] = R^{-1}B^T P$ and P is the solution of Eq. (27). The thrust u_x can be obtained by solving Eq. (14).

Computation of the Feedforward Control

Following the approach of Ref. 19, we add a feedforward control term to the nonminimum phase part so that

$$\ddot{z} = v_z + v_{ff} \tag{38}$$

$$\epsilon\ddot{\theta} = (1 + K_1)u_x \sin\alpha + (v_z + K_1 + 1)\cos\gamma - v_x \sin\gamma - a_l v^2(1 + c\alpha) \tag{39}$$

To determine the feedforward term v_{ff}, we set $x = x_c$, $\dot{x} = \dot{x}_c$, $\ddot{x} = \ddot{x}_c$, $z = z_c$, $\dot{z} = \dot{z}_c$, $\ddot{z} = \ddot{z}_c$ in Eqs. (37) and (39) to obtain

$$v_{ff} = k_3\theta^* + k_4\dot{\theta}^* \tag{40}$$

where θ^* and $\dot{\theta}^*$ are bounded solutions of

$$\epsilon\ddot{\theta}^* = (1 + K_1)u_x^* \sin\alpha^* + (\ddot{z}_c + K_1 + 1)\cos\gamma - a_l\left(\dot{x}_c^2 + \dot{z}_c^2\right)(1 + c\alpha^*) \tag{41}$$

Here $\gamma = \tan^{-1}(\dot{z}_c/\dot{x}_c)$, $\alpha^* = \theta^* - \gamma$, and u_x^* satisfies

$$\delta(\theta^*, t)u_x^{*2} + b(\theta^*, t)u_x^* + c(\theta^*, t) = 0 \tag{42}$$

where

$$\delta(\theta^*, t) = K_3\left(\frac{\sin\alpha^*}{\sqrt{\dot{x}_c^2 + \dot{z}_c^2}}\right)^2 \cos\alpha^*$$

$$b(\theta^*, t) = \frac{K_3}{\dot{x}_c^2 + \dot{z}_c^2}\sin 2\alpha^*[(\ddot{z}_c + 1)\cos\gamma] - \cos^2\alpha^*$$

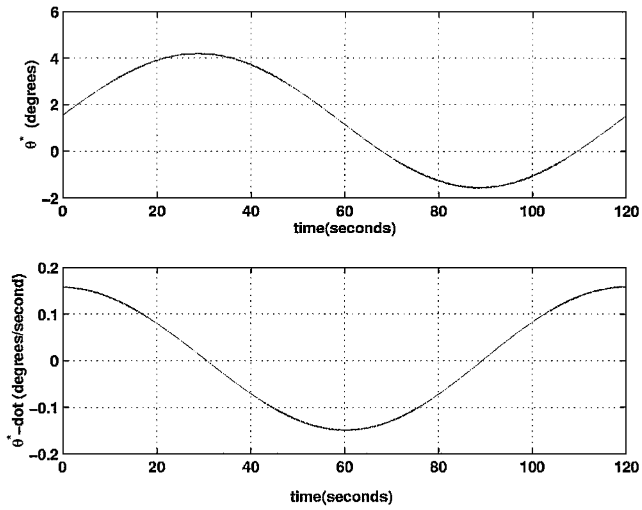

Fig. 5 Ideal driven dynamics: NOE maneuver.

$$c(\theta^*, t) = \frac{K_3}{\dot{x}_c^2 + \dot{z}_c^2}[(\ddot{z}_c + 1)\cos\gamma]^2\cos\alpha^* + [(\ddot{z}_c + 1)\cos\gamma] \times \sin\alpha^* + (\ddot{z}_c + 1)\sin\theta^* + a_d\left(\dot{x}_c^2 + \dot{z}_c^2\right)\cos\alpha^*$$

In the process of computing the bounded solutions θ^* and $\dot{\theta}^*$, we first solve Eq. (42) for u_x^* assuming that $\theta^* = \dot{\theta}^* = 0$, then this solution is used with the assumption that $\theta^* = \dot{\theta}^* = 0$ to solve Eq. (41). The updated solutions θ^* and $\dot{\theta}^*$ are then used to find the new u_x^* from Eq. (42). The preceding procedure is repeated until convergence is achieved.

For the NOE maneuver corresponding to a step command of $\dot{x}_c = 130$ m/s for the horizontal velocity and to a vertical position command given by

$$z_c(t) = \tfrac{250}{2}[1 - \cos(\pi t/60)] \tag{43}$$

the bounded solutions θ^* and $\dot{\theta}^*$ are obtained numerically using the iterative method introduced in Ref. 12. The results are shown in Fig. 5.

Tracking Control Result

The resulting tracking controller, including feedforward and feedback terms, has the form:

$$\delta(\bar{x})u_x^2 + b(\bar{x}, \bar{v})u_x + c(\bar{x}, \bar{v}) = 0 \tag{44a}$$

$$\epsilon u_m = (1 + K_1)u_x \sin\alpha + (v_z + K_1 + 1)\cos\gamma - v_x \sin\gamma - a_l v^2(1 + c\alpha) \tag{44b}$$

where

$$\delta(\bar{x}) = K_3(\sin\alpha/v)^2\cos\alpha$$

$$b(\bar{x}, \bar{v}) = \left(K_3/v^2\right)\sin 2\alpha[(v_z + 1)\cos\gamma - v_x\sin\gamma] - \cos^2\alpha$$

$$c(\bar{x}, \bar{v}) = \left(K_3/v^2\right)[(v_z + 1)\cos\gamma - v_x\sin\gamma]^2\cos\alpha + [(v_z + 1)\cos\gamma - v_x\sin\gamma]\sin\alpha + (v_z + 1)\sin\theta + v_x\cos\theta + a_d v^2\cos\alpha$$

$$v_x = \ddot{x}_c - (\beta_1/\epsilon_1)(\dot{x} - \dot{x}_c) - \left(\beta_2/\epsilon_1^2\right)(x - x_c)$$

$$v_z = \ddot{z}_c - k_1(z - z_c) - k_2(\dot{x} - \dot{z}_c) - k_3(\theta - \theta^*) - k_4(\dot{\theta} - \dot{\theta}^*)$$

The feedback controller gains are chosen as follows: $\epsilon_1 = 1$; $\beta_1 = 6$; $\beta_2 = 8$; $[k_1, k_2, k_3, k_4] = [-2.2361, -12.2245, -51.9616, -8.6262]$.

Define $e_n = (e_z, \dot{e}_z)$; $\eta_s = (\theta^*, \dot{\theta}^*)$; $\bar{\eta} = \eta - \eta_s$; and $\bar{z} = (e_n, \bar{\eta})$. The exact closed loop using the two-degrees-of-freedom controller (44) can be written as

$$\ddot{e}_x = -(\beta_1/\epsilon_1)\dot{e}_x - \left(\beta_2/\epsilon_1^2\right)e_x \tag{45}$$

$$\dot{\bar{z}} = A_c\bar{z} + \hat{g}(e_x, \dot{e}_x, e_n, \bar{\eta} + \eta_s, Y_c) - \hat{g}(0, 0, 0, \eta_s, Y_c) \tag{46}$$

Assumption 3: The incremental perturbation term in Eq. (46) satisfies the following inequality over $\|\bar{z}\|_2 < r$:

$$\|\hat{g}(0, 0, e_n, \bar{\eta} + \eta_s, Y_c) - \hat{g}(0, 0, 0, \eta_s, Y_c)\| \leq \gamma(Y_c)\|\bar{z}\| \quad (47)$$

where γ: $R \to R$ is nonnegative, continuous function satisfies the following inequality:

$$\int_0^t \gamma[Y_c(\tau)]\,\mathrm{d}\tau \leq \bar{k}t + c, \qquad t \geq 0 \quad (48)$$

for some nonnegative constants $\bar{k}$, c, where

$$\bar{k} < \frac{\lambda_{\min}(P)\lambda_{\min}(\hat{Q})}{2\lambda^2_{\max}(P)}$$

where $\hat{Q} = Q + PBR^{-1}B^TP$.

We use Assumptions 1 and 3. There exist $\epsilon_1^* > 0$ such that for all $0 < \epsilon_1 < \epsilon_1^*$ the following is true:

1) If $[e_x(0), \dot{e}_x(0), e_n(0), \eta(0)] = [0, 0, 0, \eta_s(0)]$, the output response satisfies $\eta(t) = \eta_s(t)$, $t \geq 0$, and hence $x(t) = x_c(t)$ and $z(t) = z_c(t)$, $t \geq 0$.

2) If $\|[e_x(0), \dot{e}_x(0), e_n(0), \eta(0)] - [0, 0, 0, \eta_s(0)]\|$ is small, the output response satisfies $\eta(t) - \eta_s(t) \to 0$, $x(t) - x_c(t) \to 0$, and $z(t) - z_c(t) \to 0$ as $t \to \infty$.

Proofs of these results follow from Ref. 19.

Tracking Simulations Using Feedback and Feedforward

Figure 6 shows the tracking errors corresponding to an initial state (0, 130, 0, 0, 0.0227, 0). The total tracking error is reduced from a maximum of 12 m (see Fig. 2) to a maximum of 0.12 m (see Fig. 6) after the feedforward control is added. Figure 7 shows the control signals required to execute the maneuver.

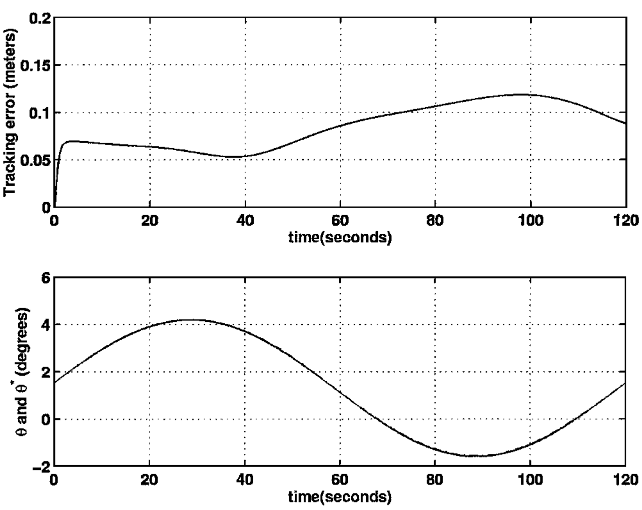

Fig. 6 NOE maneuver: feedback plus feedforward control.

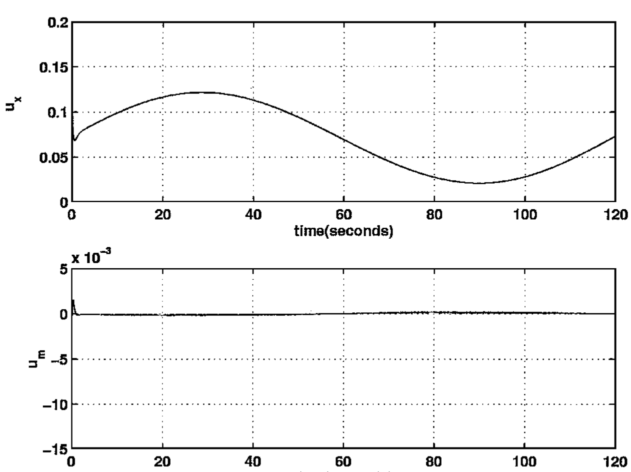

Fig. 7 Control responses: feedback plus feedforward control.

Conclusions

In this paper we studied a flight-control problem for a conventional aircraft using a longitudinal dynamics model that explicitly includes the coupling between the moment generating effectors and the aerodynamic forces. We showed that this coupling is a major source of difficulty in the design of control systems that accomplish aggressive maneuvers defined in terms of a specified inertial motion in a vertical plane. In particular, this is the source of the fact that the model is nonminimum phase if the outputs are selected as the horizontal and vertical position of the center of mass of the aircraft. The tracking problem consists of the execution of a maneuver for which the aircraft tracks a given motion in a vertical plane. We formulated the problem as a nonlinear tracking control problem. Controllers were developed for an aggressive maneuver that requires the use of a two-degrees-of-freedom controller design. An important feature of the feedforward control is that its computation requires knowledge of "future" tracking commands. We demonstrated the value of this control architecture by showing that it can achieve aggressive maneuvering with good tracking performance.

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